Investigation of the properties of the QCD-BFKL Pomeron with HERA data

talk based on

a paper with L.N. Lipatov, D.A. Ross and G. Watt

arXiv 1005.0355

and
the Pomeron/Graviton correspondence

see the talk of Chung-I Tan

H. Kowalski

DIS 2011

Newport News, 14th of April 2011

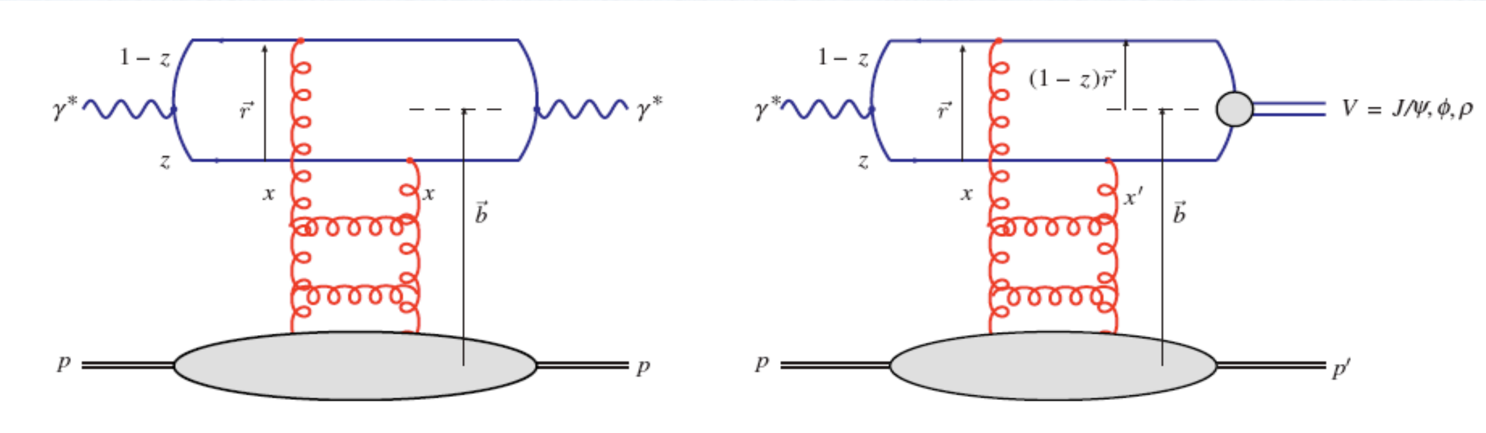
HERA - F_2 is dominated by the gluon density at low x

- ➤ **the same gluon density determines the exclusive and inclusive diffractive processes,**

  $\gamma p \Rightarrow J/\psi\ p$, $\gamma p \Rightarrow \varphi p$, $\gamma p \Rightarrow \rho p$, $\gamma p \Rightarrow Xp$,

- ➤ **universal gluon density ≡ Pomeron ?**

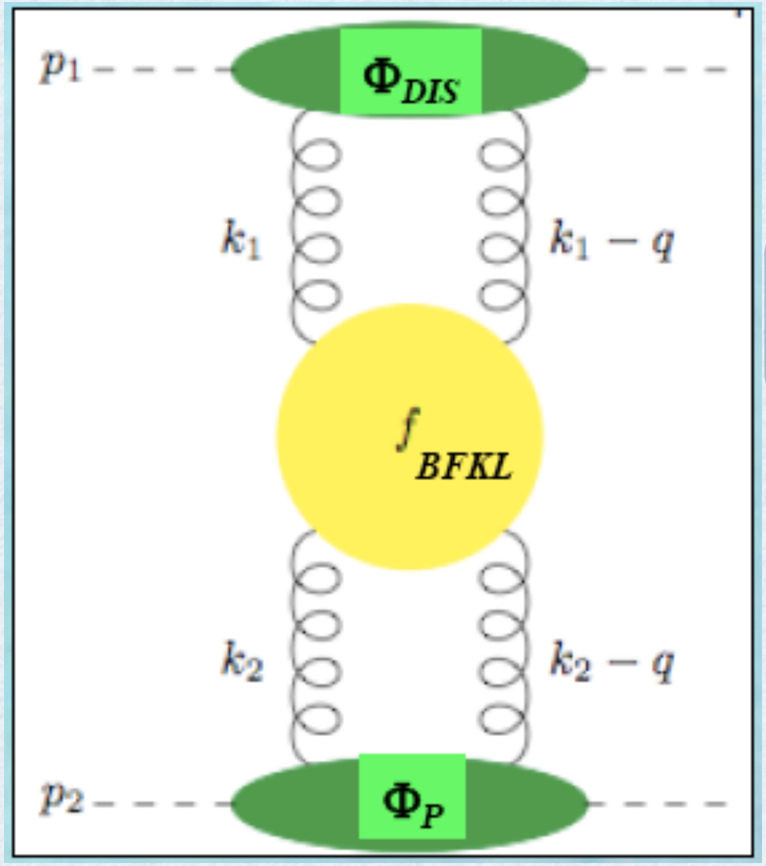
F_2 **VM, Diffraction**



clear hints for saturation, but here we concentrate on the gluon gluon interactions above the saturation region

The dynamics of Gluon Density at low x is determined by the amplitude for the scattering of a gluon on a gluon, described by the BFKL equation

$$\frac{\partial}{\partial \ln s}\mathcal{A}(s,\mathbf{k},\mathbf{k}') \;=\; \delta(k^2 - k'^2) + \int dq^2 \mathcal{K}(\mathbf{k},\mathbf{q})\mathcal{A}(s,\mathbf{q},\mathbf{k}')$$



which can be solved in terms of the eigenfunctions of the kernel

$$\int dk'^2 \mathcal{K}(\mathbf{k},\mathbf{k}')f_\omega(\mathbf{k}') \;=\; \omega f_\omega(\mathbf{k})$$

in LO, with fixed α_s

$$f_\omega(\mathbf{k}) \;=\; \left(k^2\right)^{i\nu - 1/2}$$
$$\omega \;=\; \alpha_s \chi_0(\nu)$$

prevailing intuition (based on DGLAP) - gluon are a gas of particles

BFKL leads to a richer structure - basic feature: oscillations

Properties of the BFKL Kernel

Quasi-locality

$$\mathcal{K}(\mathbf{k},\mathbf{k}') \;=\; \frac{1}{kk'}\sum_{n=0}^{\infty} c_n \delta^{(n)}\left(\ln(\mathbf{k}^2/\mathbf{k}'^{\,2})\right)$$

$$c_n = \int_0^{\infty} dk'^{\,2}\mathcal{K}(\mathbf{k},\mathbf{k}')\frac{k}{k'}\frac{1}{n!}\left(\ln(\mathbf{k}^2/\mathbf{k}'^{\,2})\right)^n$$

Similarity to the Schroedinger equation

$$k\int dk'^{\,2}\mathcal{K}(\mathbf{k},\mathbf{k}')f_\omega(\mathbf{k}') \;=\; \sum_{n=0}^{\infty} c_n\left(\frac{d}{d\ln(\mathbf{k}^2)}\right)^n \bar{f}_\omega(\mathbf{k}) \;=\; \omega\bar{f}_\omega(\mathbf{k})$$

Characteristic function

$$k\int dk'^{\,2}\mathcal{K}(\mathbf{k},\mathbf{k}')f_\omega(\mathbf{k}') \;=\; \chi\left(-i\frac{d}{d\ln k^2},\alpha_s(k^2)\right)\bar{f}_\omega(k) \;=\; \omega\bar{f}_\omega(k)$$

BFKL amplitude

$$A(s,\mathbf{k_1},\mathbf{k_2}) \sim \int d\nu \left[\frac{\mathbf{k_1^2}}{\mathbf{k_2^2}}\right]^{i\nu} s^{\bar{\alpha}_s \chi(\nu)}$$

$$\bar{\alpha}_s = C_A \frac{\alpha_s}{\pi}$$

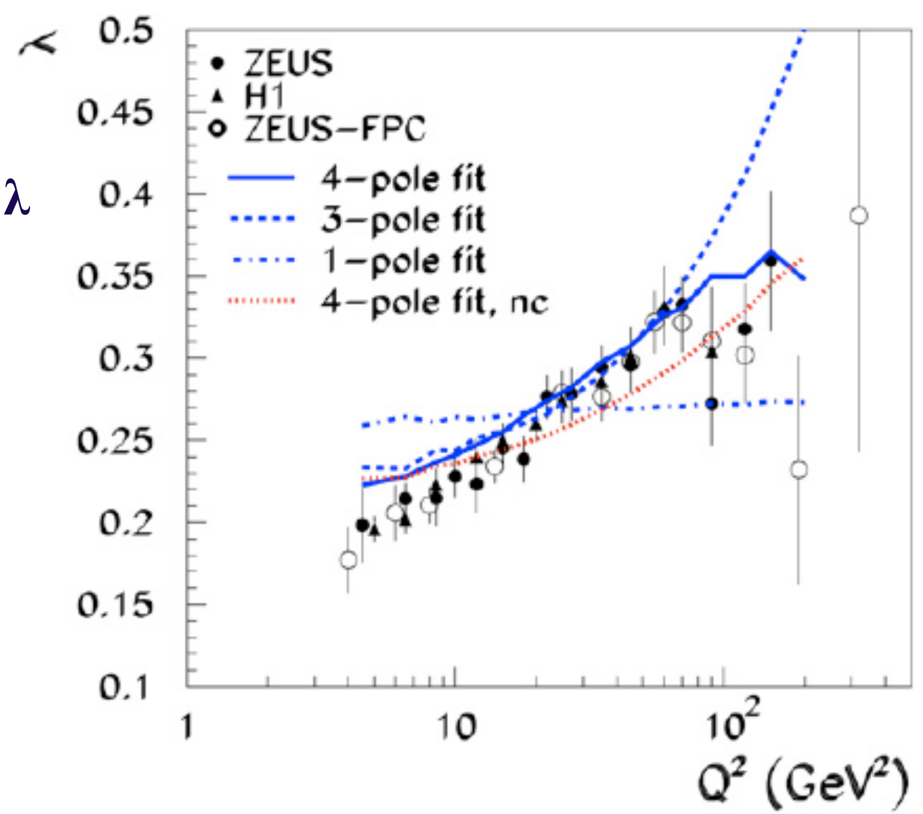
Diffusion approximation

$$\chi(\nu) = 4\ln(2) - 7\zeta(3)\nu^2 + ...$$

$$\mathcal{A}(s,t,\mathbf{k_1},\mathbf{k_2}) \sim \int d\nu\, s^{1+\overline{\alpha_s}\left(4\ln(2)-7\zeta(3)\nu^2+\cdots\right)} e^{i\nu(ln(\mathbf{k_2})-ln(\mathbf{k_1}))}$$

BFKL eq., with fixed α_s, predicts $F_2 \sim (1/x)^{\omega}$ with $\omega \sim$ constant with Q^2, $\omega \sim 0.5$ in LO and $\omega \sim 0.3$ in NLO
Therefore, the prevailing opinion was that the BFKL analysis is not applicable to HERA data.

The rate of rise λ
$F_2 \sim (1/x)^{\lambda}$



First hints that in BFKL λ can be substantially varying with Q^2 was given in PL 668 (2008) 51 by EKR

Lipatov 86 & EKR 2008: BFKL solutions with the running α_s are substantially different from solutions with the fixed α_s.

in NLO, with running α_s, BFKL frequency ν becomes k-dependent, $\nu(k)$

$$\alpha_s(k^2)\chi_0(\nu(\mathbf{k})) + \alpha_s^2(k^2)\chi_1(\nu(\mathbf{k})) \ = \ \omega$$

ν has to become a function of k because ω cannot depend on k

GS resummation applied

evaluation in diffusion ($\nu \approx 0$) or semiclassical approximation ($\nu > 0$)

For sufficiently large k, there is no longer a real solution for ν.

The transition from real to imaginary $\nu(k)$ singles out a special value of

$k = k_{crit}$, with $\nu(k_{crit}) = 0$.

The solutions below and above this critical momentum k_{crit} have to match. This fixes the phase of ef's.

Near $k=k_{crit}$, the BFKL eq. becomes the Airy eq. which is solved by the Airy eigenfunctions

$$k\,f_\omega(k) = \bar{f}_\omega(k) = \mathrm{Ai}\left(-(\frac{3}{2}\phi_\omega(k))^{\frac{2}{3}}\right)$$

with

$$\phi_\omega(k) = 2\int_k^{k_{\mathrm{crit}}} \frac{d\,k'}{k'}|\nu_\omega(k')|$$

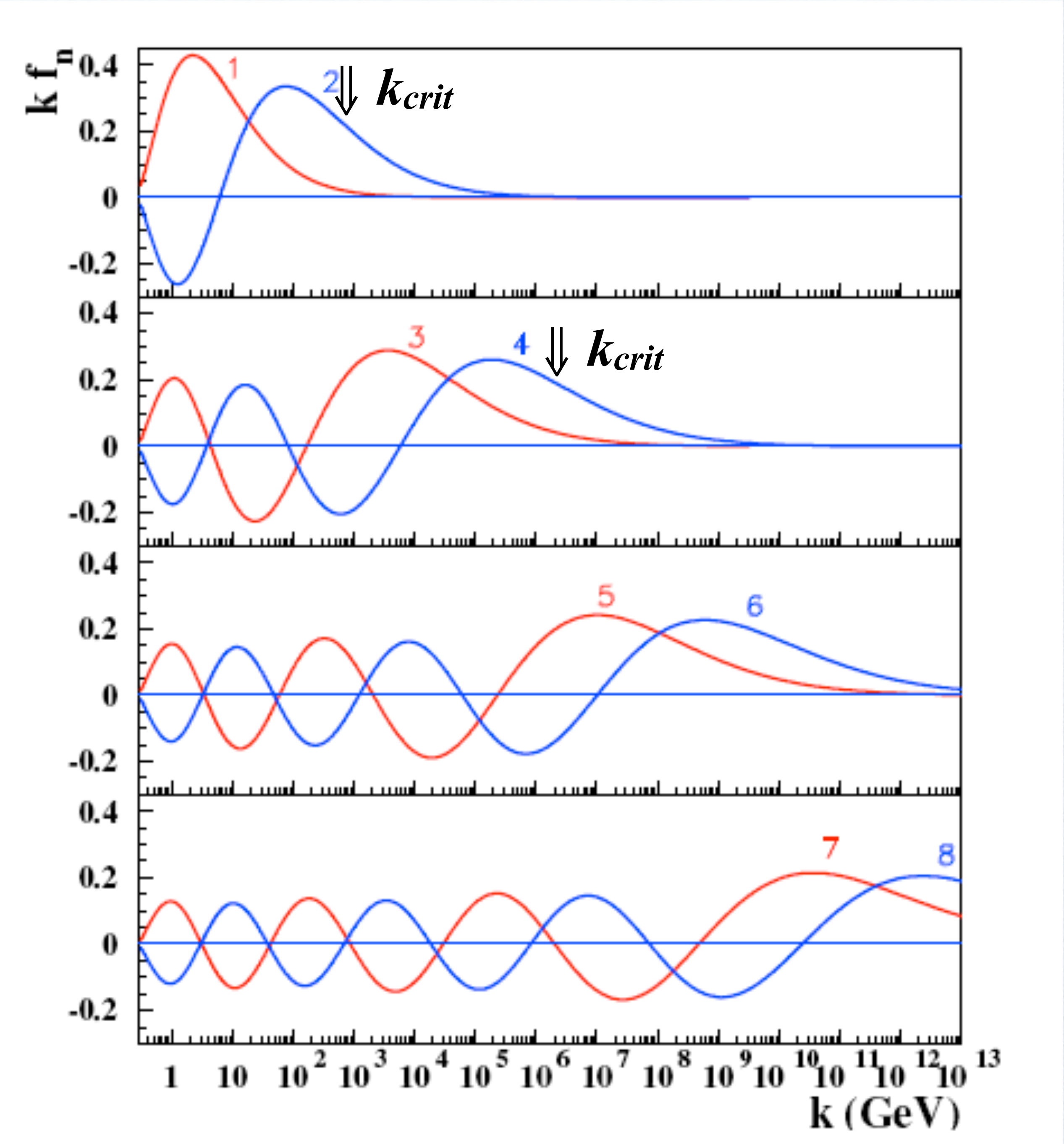
for $k<<k_{crit}$ the Airy function has the asymptotic behaviour

$$k\,f_\omega(k) \sim \sin\left(\phi_\omega(k) + \frac{\pi}{4}\right)$$

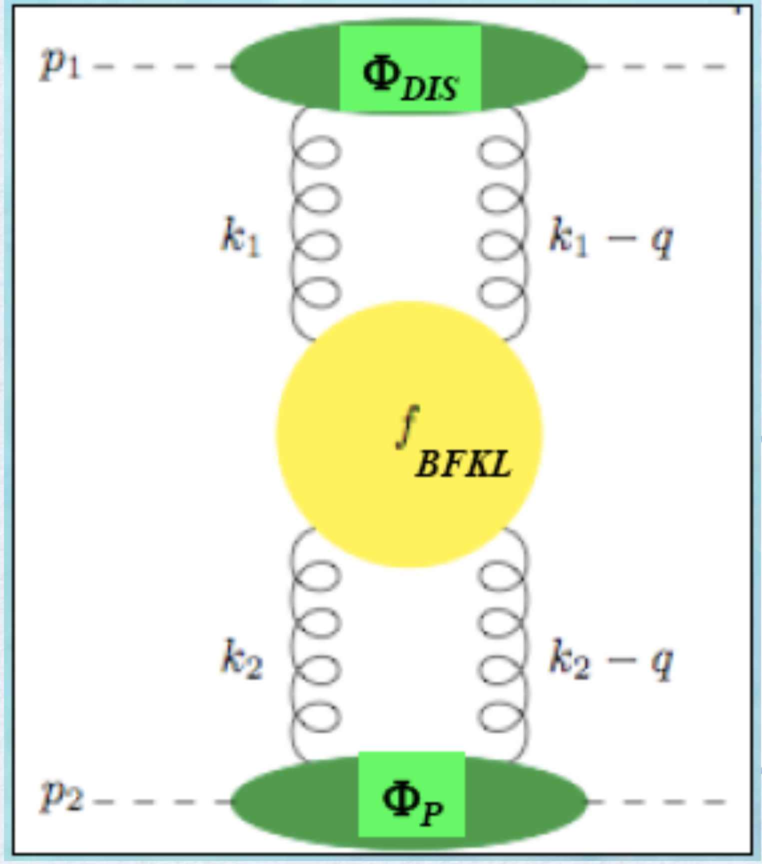
The two fixed phases at $k=k_{crit}$ and at $k=k_0$ (near Λ_{QCD}) lead to the quantization condition

$$\phi_\omega(k_0) = \left(n - \frac{1}{4}\right)\pi + \eta\,\pi$$

The first eight eigenfunctions determined at $\eta=0$



Comparison with HERA data



Discreet Pomeron Green function

$$\mathcal{A}(\mathbf{k},\mathbf{k}') = \sum_{m,n} f_m(\mathbf{k}) \mathcal{N}^{-1}_{mn} f_n(\mathbf{k}') \left(\frac{s}{kk'}\right)^{\omega_n}.$$

Integrate with the photon and proton impact factors

$$\mathcal{A}^{(U)}_n \equiv \int_x^1 \frac{d\xi}{\xi} \int \frac{dk}{k} \Phi_{\mathrm{DIS}}(Q^2,k,\xi) \left(\frac{\xi k}{x}\right)^{\omega_n} f_n(\mathbf{k})$$

$$\mathcal{A}^{(D)}_m \equiv \int \frac{dk'}{k'} \Phi_p(k') \left(\frac{1}{k'}\right)^{\omega_m} f_m(\mathbf{k}').$$

$$F_2(x,Q^2) = \sum_{m,n} \mathcal{A}^{(U)}_n \mathcal{N}^{-1}_{nm} \mathcal{A}^{(D)}_m$$

Proton impact factor

$$\Phi_p(\mathbf{k}) \;=\; A\,k^2 e^{-bk^2}$$

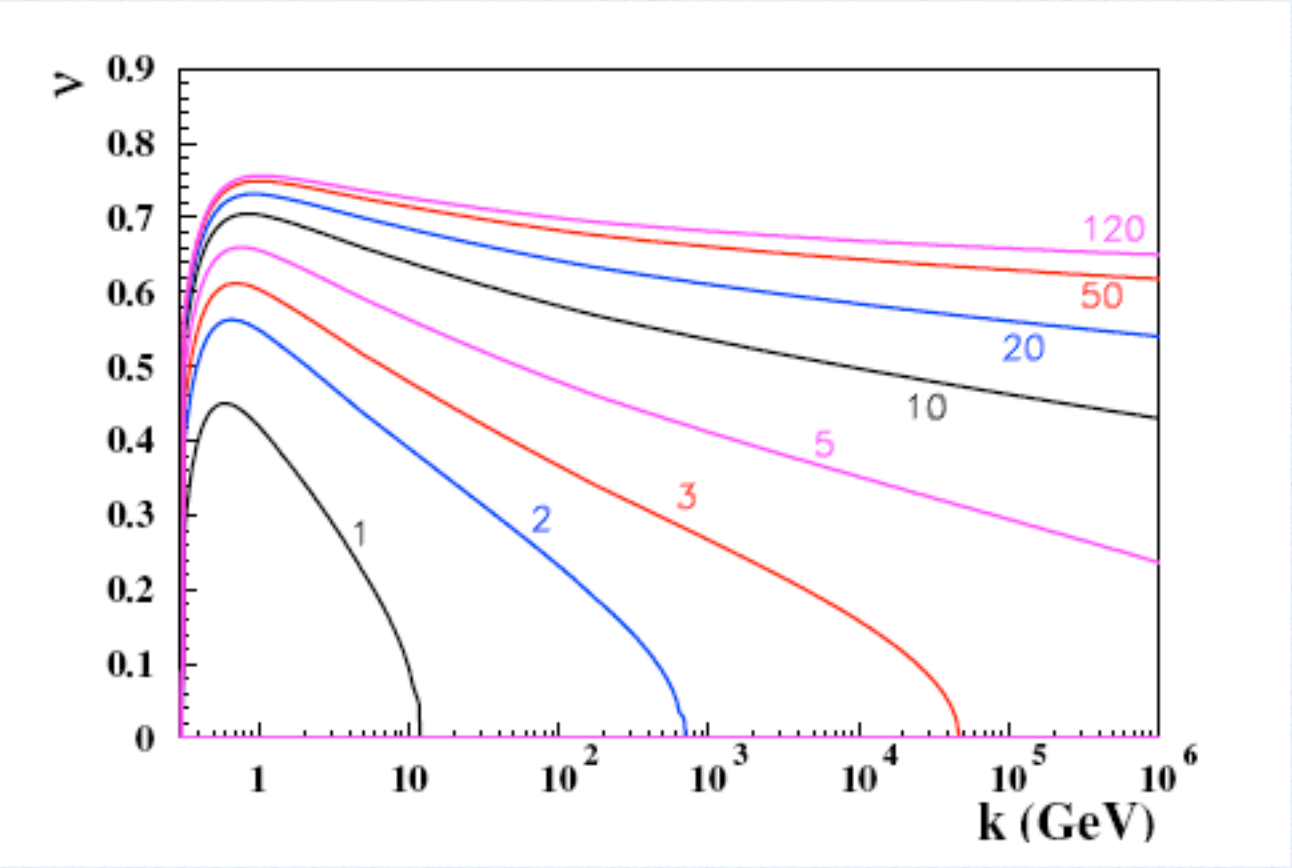
The fit is not sensitive to the particular form of the impact factor.

The support of the proton impact factor is much smaller than the oscillation period of f_n and because the frequencies ν have a limited range

➤ many eigenfunctions have to contribute and η has to be a function of n

$$\eta \;= \eta_0 \left(\frac{n-1}{n_{\max}-1} \right)^{\kappa}$$

The frequencies $\nu(k)$



Music analogy:
eigenfunctions are tones with modulated frequencies

The qualities of fits for various numbers of eigenfunctions

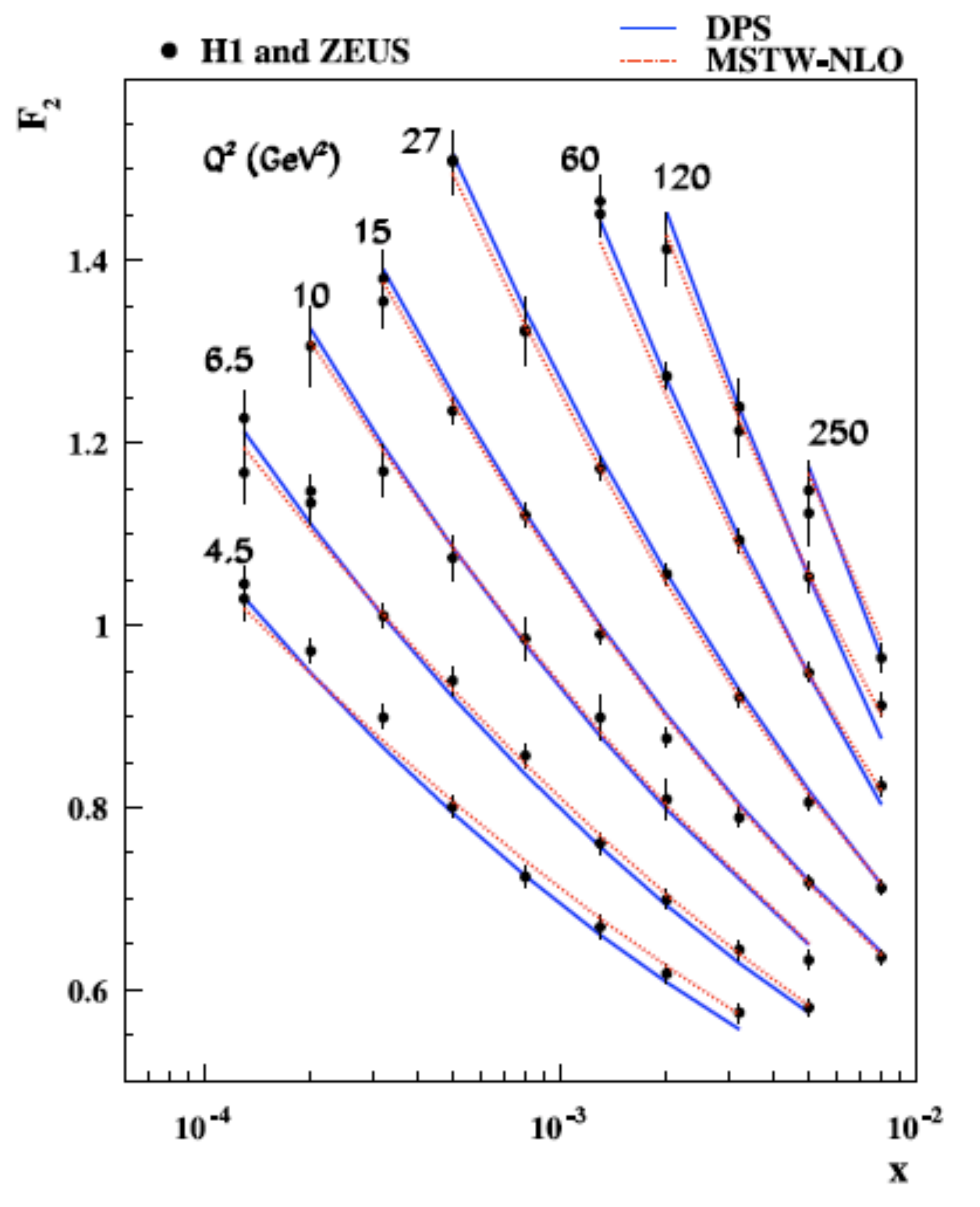
(a) Fits with cuts of $Q^2 > 4$ GeV^2 and $x < 0.01$

$n_{\max}$	$\chi^2/N_{\mathrm{df}}(x < 0.01)$	$\chi^2/N_{\mathrm{dat}}(x < 0.001)$	κ	A	b
1	$9792/125 = 78.3$	$2123/43 = 49.4$	–	156	30.0
5	$349.8/125 = 2.80$	$88.8/43 = 2.07$	3.78	$3.1 \cdot 10^6$	78.0
20	$286.5/125 = 2.29$	$83.3/43 = 1.94$	0.96	632	15.8
40	$193.3/125 = 1.55$	$54.9/43 = 1.28$	0.84	2315	23.2
60	$163.3/125 = 1.31$	$44.8/43 = 1.04$	0.78	3647	25.6
80	$156.5/125 = 1.25$	$43.5/43 = 1.01$	0.73	3081	24.4
100	$149.1/125 = 1.19$	$41.3/43 = 0.96$	0.69	2414	22.8
120	$143.7/125 = 1.15$	$39.2/43 = 0.91$	0.66	2041	21.8

➤ new data are crucial for finding the right solution

the differences in the fit qualities would be negligible if the errors where more than 2-times larger

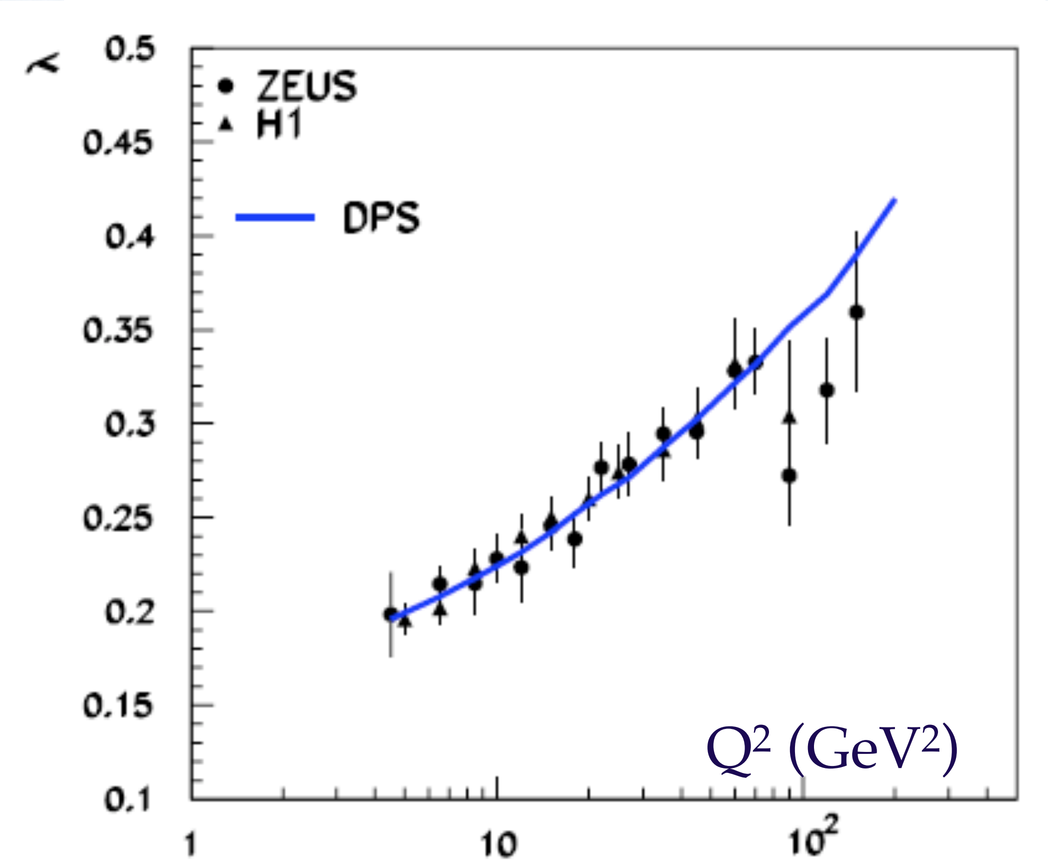
The final fit performed with 120 ef's and 30 overlaps and 5 flavours



χ^2/N_{df}	κ	A	b
154.7 /125	0.65	1660	20.6

The rate of rise λ

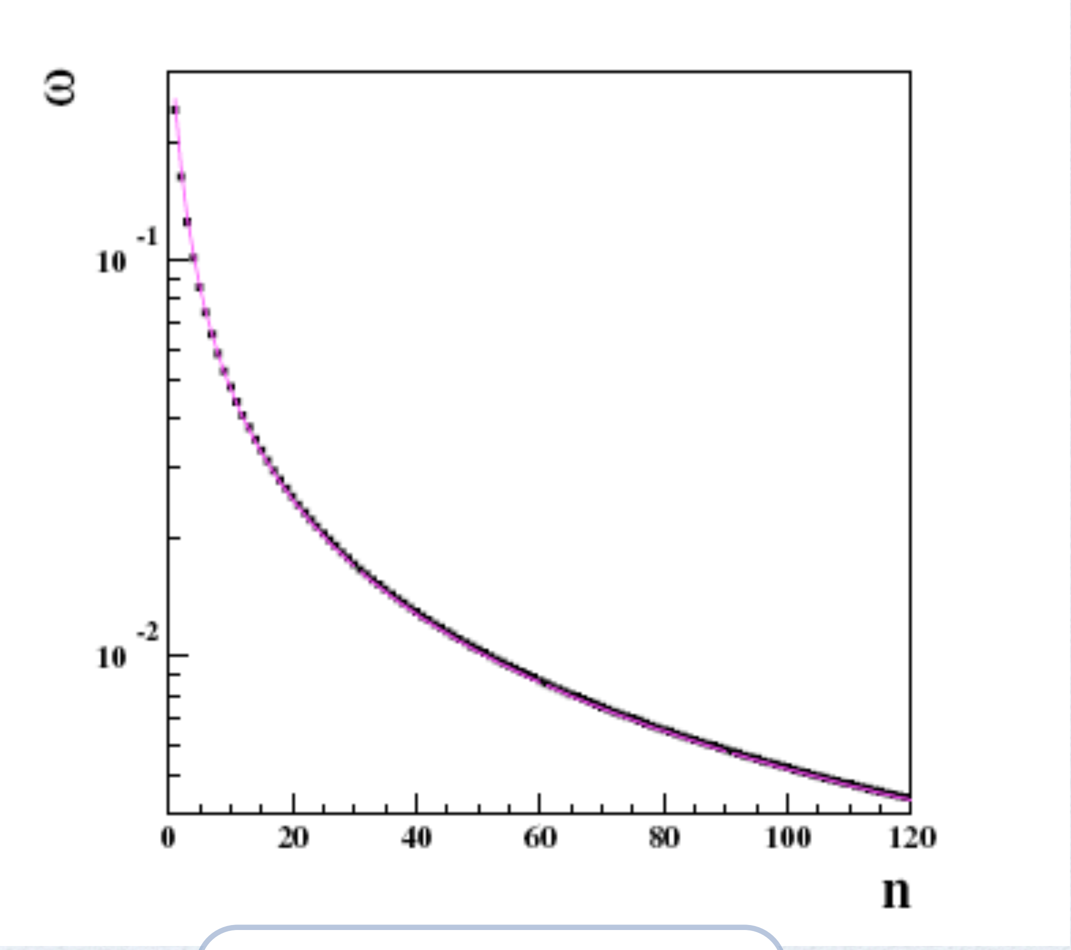
$F_2 \sim (1/x)^{\lambda}$



The first successful pure BFKL description of the λ plot.

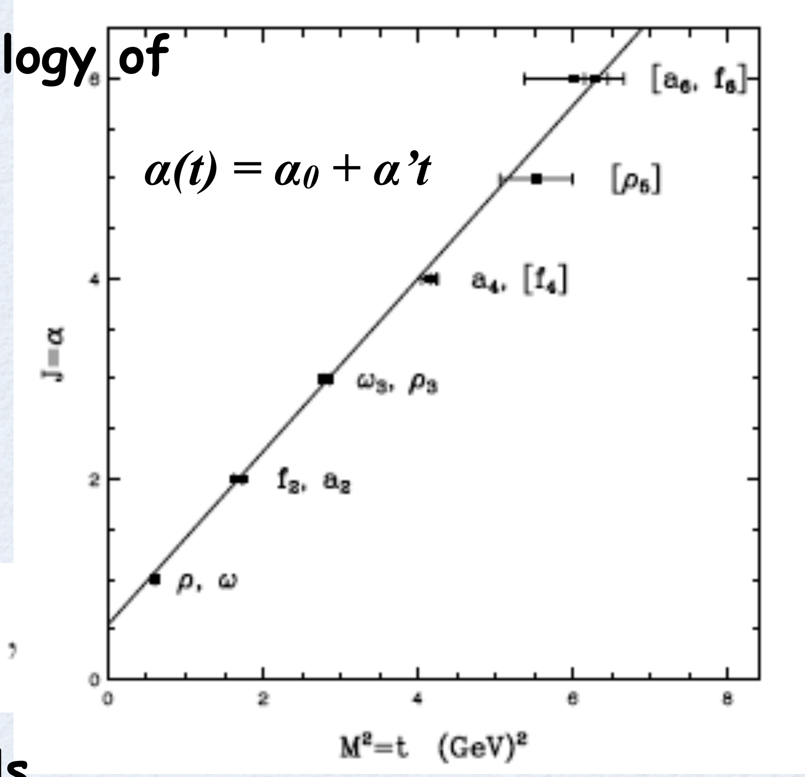
For many years it was claimed that BFKL analysis was not applicable to HERA data because of the observed substantial variation of λ with Q^2

Eigenvalues ω



$$\omega_n \approx \frac{0.5}{1 + 0.95\, n}$$

Pomeron - Graviton Correspondence



String theory emerged out of phenomenology of hadron-hadron scattering - Dolan-Horn-Schmid duality

$$\sum_r \frac{g_r^2(t)}{s-(M_r - i\Gamma_r)^2} \simeq \beta(t)(-\alpha' s)^{\alpha(t)}$$

▶ **Veneziano amplitude**

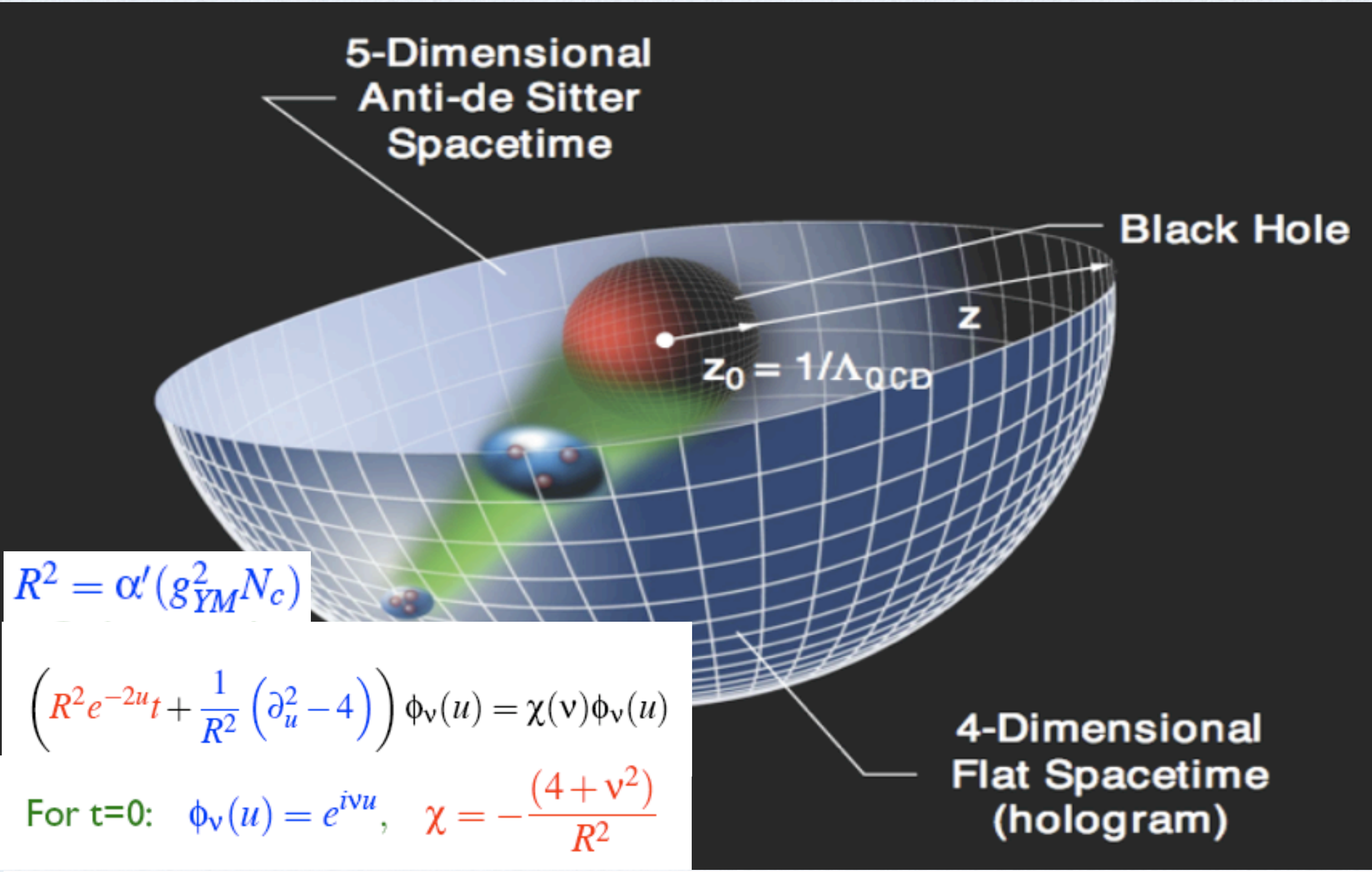
$$A_{\pi^+\pi^-\to\pi^+\pi^-}(s,t) = g_o^2 \frac{\Gamma[1-\alpha_\rho(t)]\Gamma[1-\alpha_\rho(s)]}{\Gamma[1-\alpha_\rho(s)-\alpha_\rho(t)]},$$

▶ **generalization to dual resonance models, Veneziano amplitude for the pomeron trajectory has a pole for** $s=t=0$ ***with*** $J=2$

▶ **starting point for a theory of quantum gravity**

Maldacena Conjecture: (N=4 SUSY YM QCD) = (CFT in $ADS_5 \times S^5$)

Pomeron in ADS, Brower, Polchinski, Strassler, Tan, 2006



$$R^2 = \alpha'(g_{YM}^2 N_c)$$

$$\left(R^2 e^{-2u} t + \frac{1}{R^2}\left(\partial_u^2 - 4\right)\right)\phi_\nu(u) = \chi(\nu)\phi_\nu(u)$$

For t=0: $\phi_\nu(u) = e^{i\nu u}$, $\chi = -\frac{(4+\nu^2)}{R^2}$

$u=\ln(z_0/z)$ in ADS corresponds to $\ln(k/k_0)$ in BFKL

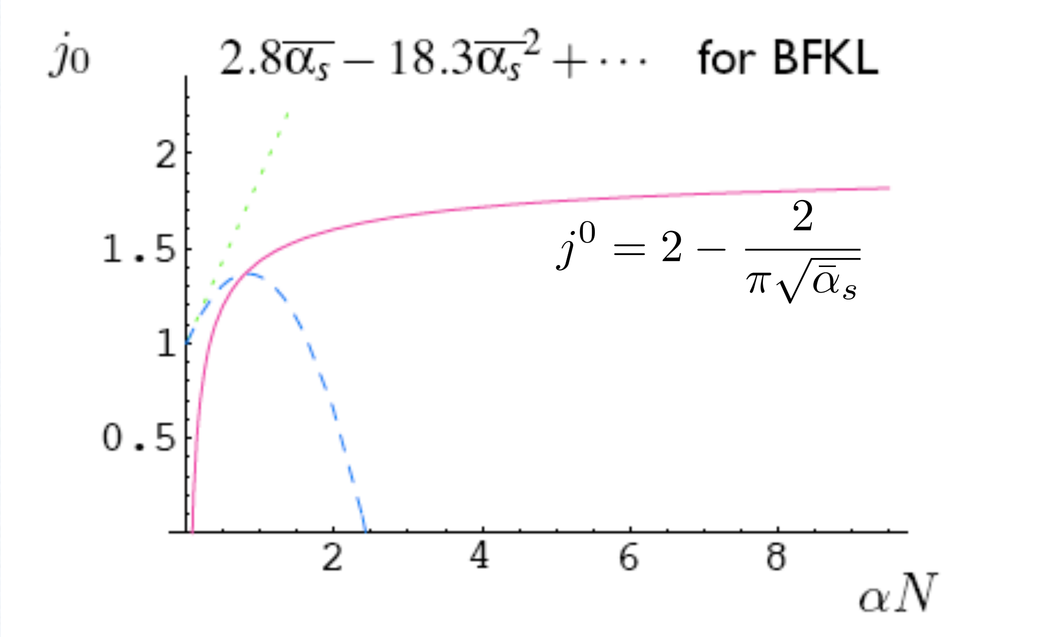
Pomeron and Gauge/String Duality

Brower, Polchinski, Strassler, and Tan, hep-th/0603115

Pomeron is a coherent color-singlet object, build from gluons, with universal properties; it is the object which is exchanged by any pair of hadrons that scatter at high energies.

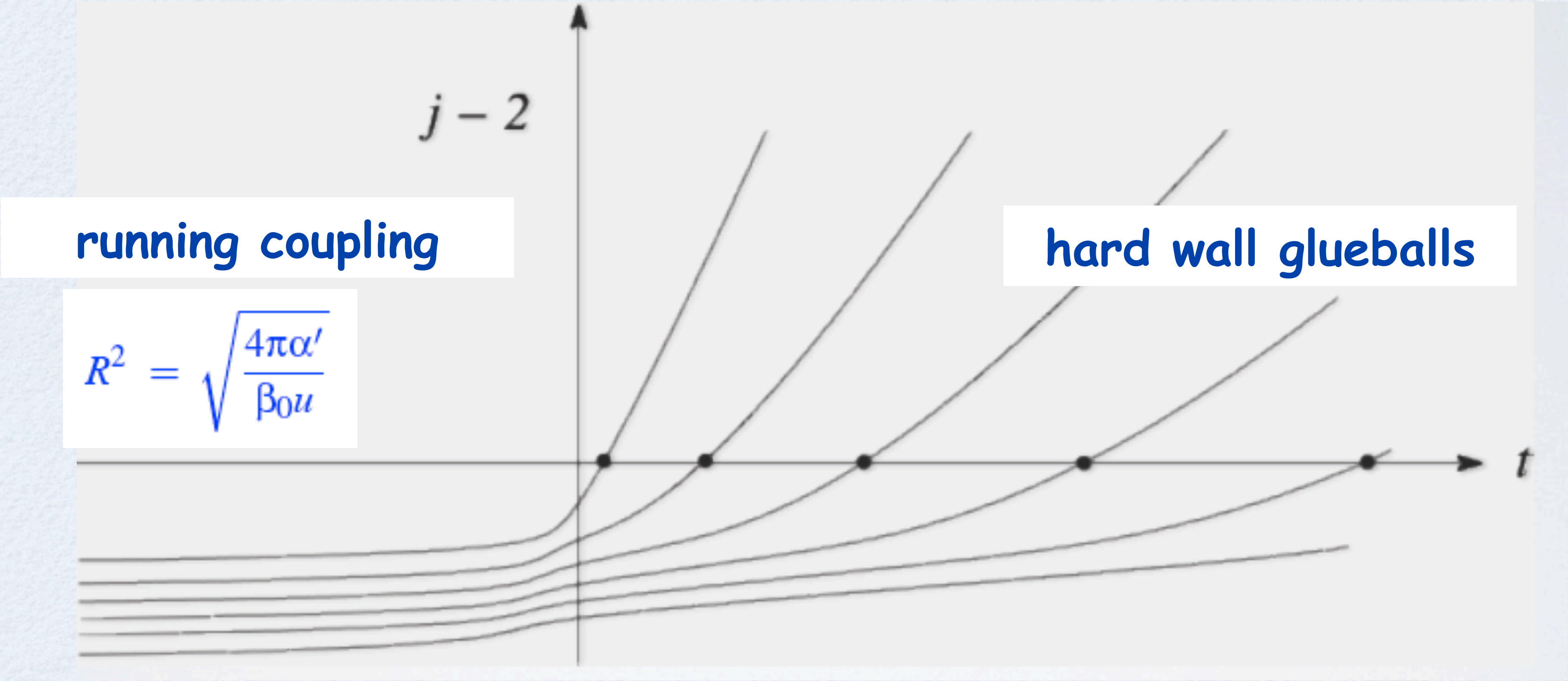
$$j^0 = 2 - \frac{2}{\pi\sqrt{\bar{\alpha}_s}}$$ in ADS5 and in N=4 Super YM

Kotikov, Lipatov, Onishchenko, Velizhanin, Physt. Lett. B 632, 754 (2006)



$$\left(\overline{\alpha_s} = \frac{\alpha_s N_C}{\pi}\right)$$

Pomeron Regge trajectories in ADS



String-Gauge Dual Description of Deep Inelastic Scattering at Small-x

arXiv: 1007.2259v2, Sept 2010

Richard C. Brower*, Marko Djurić†, Ina Sarčević‡§ and Chung-I Tan¶

direct term

reflected term

$$F_2(x,Q^2) = \frac{g_0^2 \rho^{3/2}}{32\pi^{5/2}} \int dz dz' P_{13}(z,Q^2) P_{24}(z')(zz'Q^2)\, e^{(1-\rho)\tau} \left(\frac{e^{-\frac{\log^2 z/z'}{\rho\tau}}}{\tau^{1/2}} + \mathcal{F}(z,z',\tau) \frac{e^{-\frac{\log^2 zz'/z_0^2}{\rho\tau}}}{\tau^{1/2}} \right)$$

$$P_{13}(z) \approx C\delta(z - 1/Q).$$

$$P_{24}(z') \approx \delta(z' - 1/Q').$$

$$e^{(1-\rho)\tau} \sim (1/x)^{1-\rho}$$

$$\mathcal{F}(z,z',\tau) = 1 - 2\sqrt{\rho\pi\tau}\, e^{\eta^2} erfc(\eta), \qquad \eta = \frac{-\log \frac{zz'}{z_0^2} + \rho\tau}{\sqrt{\rho\tau}}.$$

reflected term (model dependent) corresponds to the phase condition in KLRW

fitted variables,

g_0, ρ, z_0, Q'

in KLRW, ρ is predicted

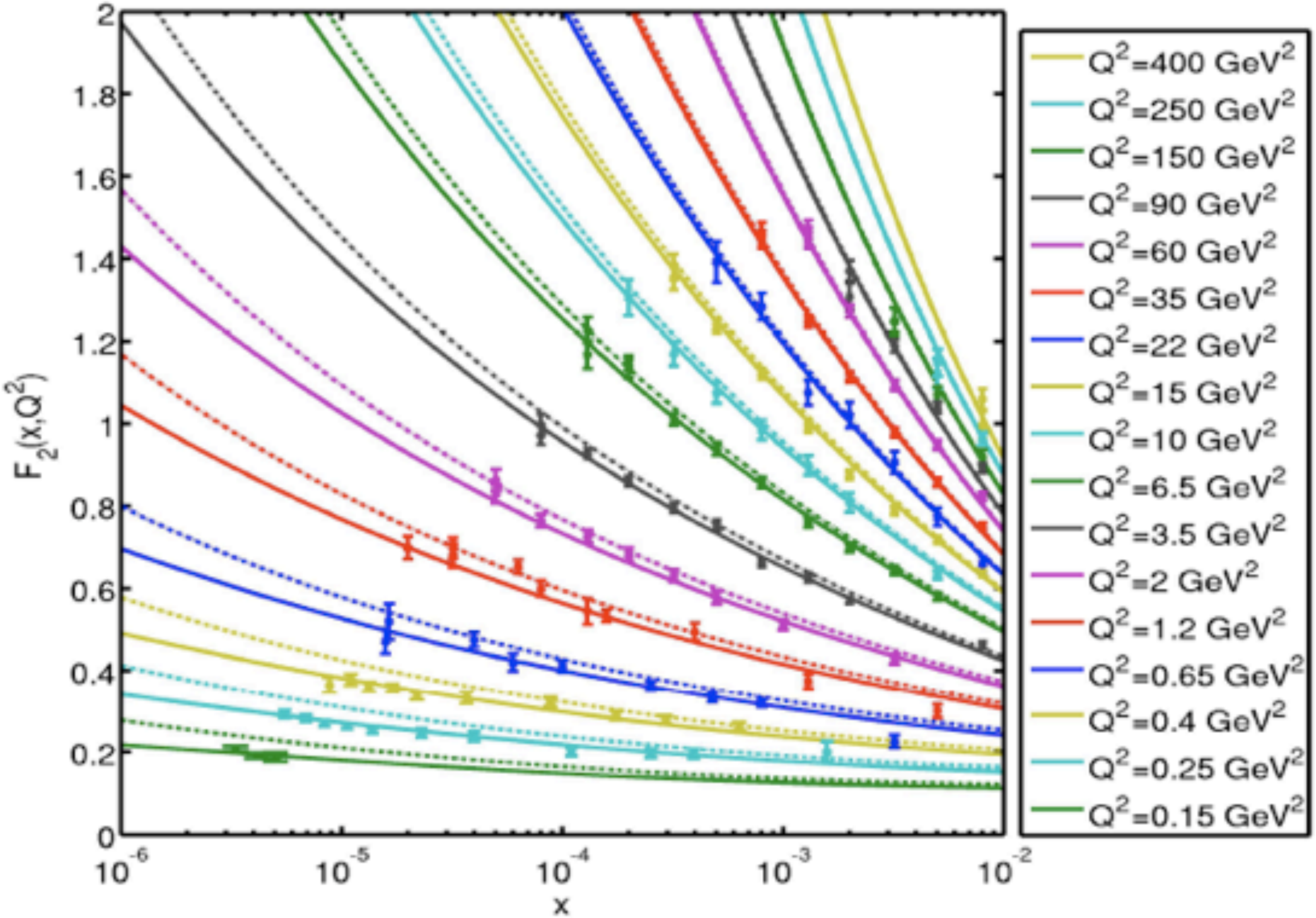


Figure 9: Fit to the combined H1-ZEUS small-x data for $F_2(x, Q^2)$ by a hard-wall eikonal treatment. We have exhibited both the hard-wall single Pomeron fits, (in dashed lines), and the hard-wall eikonal, (in solid lines), together for a better visual comparison. The fit include 249 data points, with $x < 10^{-2}$, and 34 Q^2 values, ranging from 0.1 GeV^2 to 400 GeV^2. Only data set for 17 Q^2 values are shown.

Summary and Outlook

Since the beginning of particle physics, high energy behavior of scattering amplitudes was expected to give basic insight into the nature of strong forces. (at HE, time dilatation slows down the dynamics of physical processes)

Two different basic approaches: the Discrete-BFKL-Pomeron and ADS-closed-string-Pomeron are describing HERA F_2 data very well.

Will striking similarities between the two approaches give insight into the connection between QCD and Gravitation? Into the confinement problem?

Precise measurement at future ep and eA could provide crucial data:

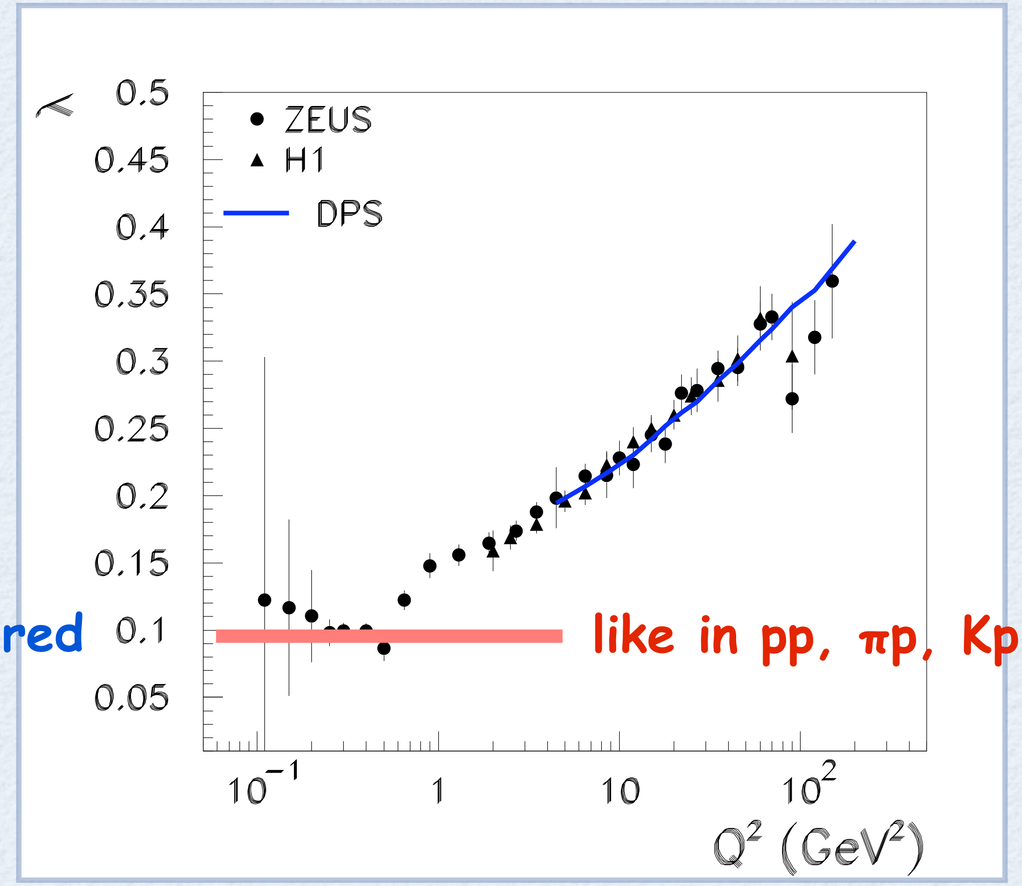
1) exclusive diffractive processes ⇒ measurements of $\alpha(t)$ - EIC

2) F_2 and exclusive diffraction at highest possible energies - LHeC

Back up slides

Back up slides

Transition to the saturation and confinement regions

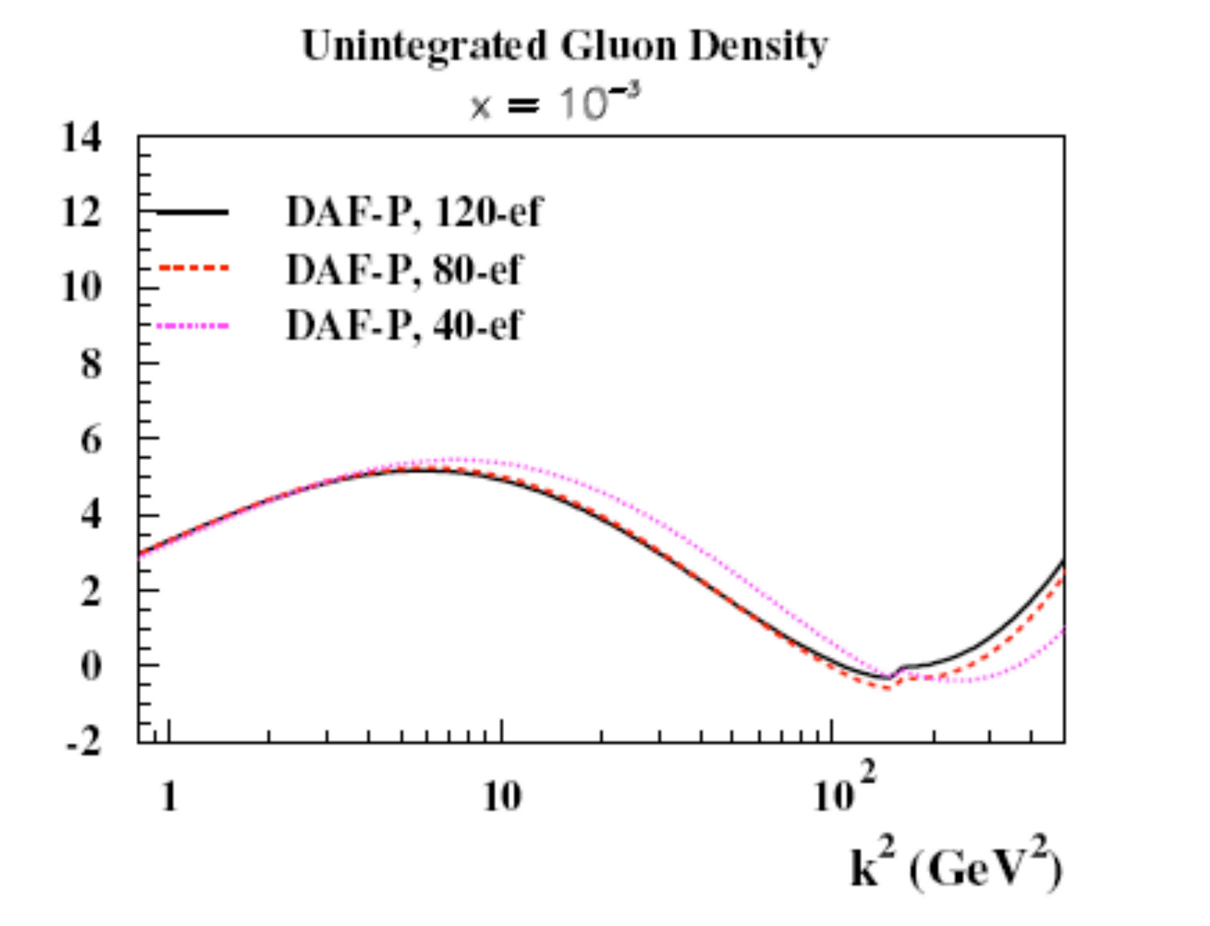


precision data
at low Q^2 required

like in pp, πp, Kp scattering

evaluate triple pomeron vertex with DPS, at $t \neq 0$, apply it in the saturation region, i.e at low Q^2 , and to elastic pp scattering

High energy behaviour of $pp, \pi p, Kp$ and γp shows universal properties ☞ get insight into confinement?



Unintegrated Gluon Density
x = 10^-3
DAF-P, 120-ef
DAF-P, 80-ef
DAF-P, 40-ef
14
12
10
8
6
4
2
0
-2
1
10
10^2
k^2 (GeV^2)

why so many eigenfunctions ?

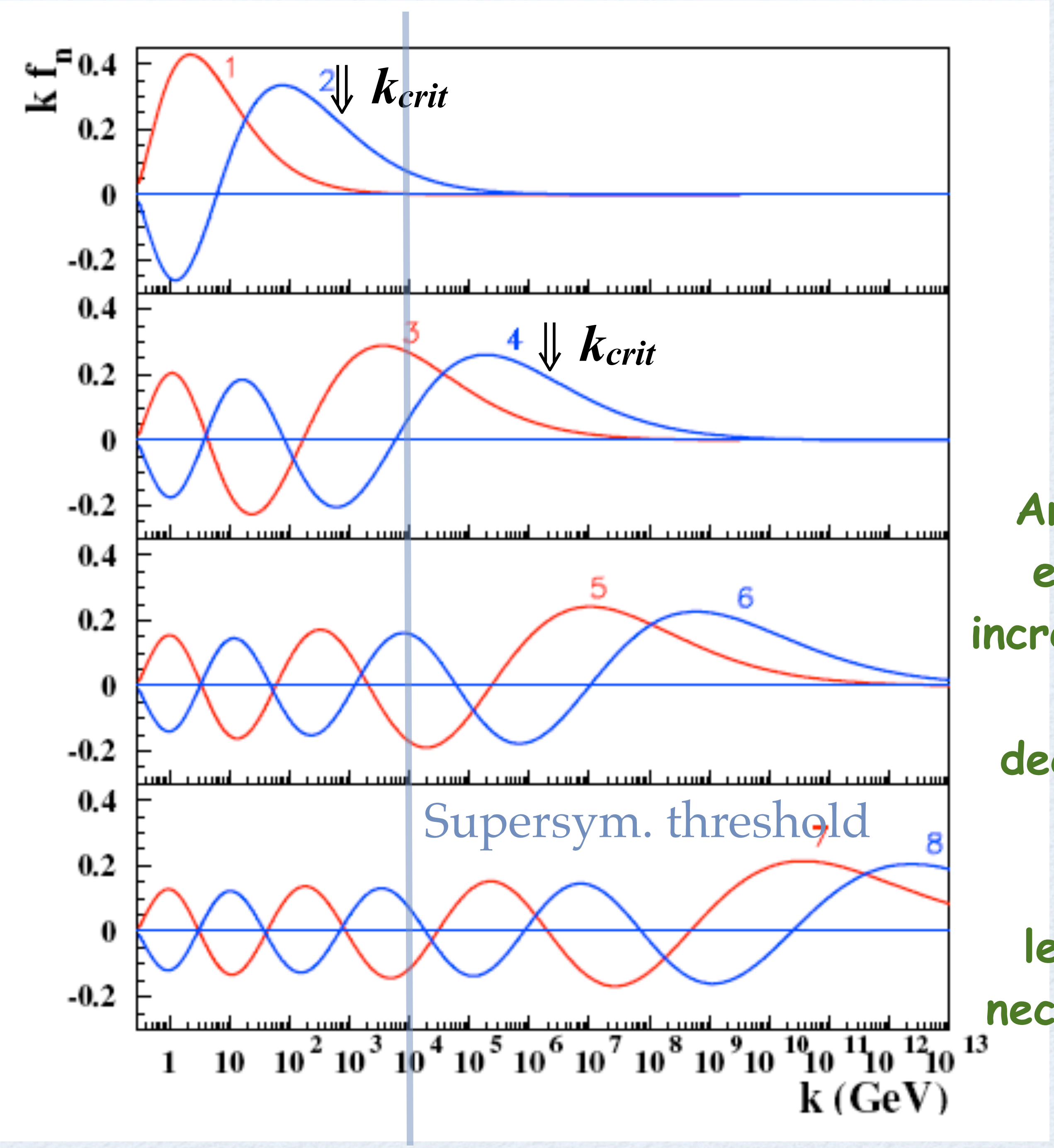
because the contribution of large n ef's is only weakly suppressed

enhancement by $(1/x)^{\omega}$ is not very large because

$\omega_1 \approx 0.25, \quad \omega_5 \approx 0.1, \quad \omega_{10} \approx 0.05$

suppression of large n contribution only by the normalization condition $\sim 1/\sqrt{n}$

The first eight eigenfunctions determined at η=0



Are BSM effects increasing ν ? and decreasing k_{crit} ?

less ef's necessary ?

Quasi-locality of the kernel

$$\mathcal{K}(\mathbf{k},\mathbf{k}') \;=\; \frac{1}{kk'}\sum_{n=0}^{\infty} c_n \delta^{(n)}\left(\ln(\mathbf{k}^2/\mathbf{k}'^2)\right),$$

and of the Green function